

# Multi-Dimensional Simulations of Pair-Instability Supernovae

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## Abstract

We present preliminary results from multidimensional numerical studies of pair instability supernova (PSN), studying the fluid instabilities that occur in multiple spatial dimensions. We use the new radiation-hydrodynamics code, CASTRO, and introduce a new mapping procedure that defines the initial conditions for the multidimensional runs in such a way that conservation of physical quantities is guaranteed at any level of resolution.

**Keywords:** Stellar evolution, Massive star, Pair instability supernovae

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## 1. Introduction

The first stars that formed after the big bang may have a characteristic mass scale of around hundred solar masses ( $M_{\odot}$ ) [1, 2, 3]. The stars with initial mass between  $140 M_{\odot}$  and  $260 M_{\odot}$  end their lives in a very powerful explosion, called a pair-instability supernova (PSN) [4, 5, 6]. During the evolution of those massive stars, after central carbon burning, the core of the star reaches a sufficiently high temperature that electron and positron pairs can be produced. At this time, radiation energy turns into rest mass for these pairs. It softens the adiabatic index  $\gamma$  of the gas below the critical value of  $4/3$  and triggers a rapid contraction that leads to explosive burning of oxygen and silicon. The energy released raises the pressure enough to turn the contraction around into a energetic thermonuclear explosion ( $\sim 3 - 100 \times 10^{51}$  erg). These supernovae may play an important role in the synthesis of heavy elements [6, 7] and their energetic feedback to their surroundings can affect the structure and evolution of the early universe [8, 9]. Although several PSN candidates have been observed [10, 11], there are still many discrepancies between models and observations [12]. The current theoretical models for PSN are all based on one-dimensional calculations [6, 7]; until now, no multidimensional simulations have scarce. Here we study how multi-dimensional fluid instabilities affect the mixing of elements and possibly even the overall nucleosynthesis and energetics of PSN.

In this paper, we first introduce our numerical approach in § 2. Then, in § 3, we introduce a new mapping procedure that defines the initial conditions for the multidimensional simulations in such a way that conservation of physical quantities is guaranteed at any resolution. We discuss the results of our simulation in § 4 and present our conclusions in § 5.

## 2. Numerical Approach

We start our simulations using one-dimensional models obtained from the KEPLER code [13], spherically symmetric Lagrangian code that followed the evolution of a  $150 M_{\odot}$  star up to ten seconds before maximum compression. Then we map the resulting one-dimensional profiles into 2D and 3D to serve as the initial conditions for CASTRO [14]. We then evolve the simulation for about one hundred seconds. This is period during which thermonuclear burning releases almost all of the energy of the explosion.

CASTRO [14] is a new, massively parallel, multi-dimensional Eulerian AMR radiation-hydrodynamics code for astrophysical applications. Time integration of the hydrodynamics equations is based on a higher-order, unsplit Godunov scheme. Block-structured adaptive mesh refinement (AMR) and sub-cycling in time enable the use of high spatial resolution where it is most needed. We refer the readers to [14, 15] for details.

CASTRO allows the user to specify their equation of state, reaction network, and method for calculating self-gravity. For the simulations presented here, we use the Helmholtz equation of state [16]. Our reaction network

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contains 19 isotopes and 86 reaction rates [13]. This network is sufficient for us to model the burning processes and their energy release to sufficient accuracy. We use the monopole approximation for self-gravity.

### 3. Mapping of Initial Model in Multi-D

Since our initial model will be in a state close to hydrostatic equilibrium, we need to be careful how we map the 1D spherically symmetric data given on a non-uniform Lagrangian grid to a multidimensional Eulerian grid. Here we present a method that numerically conserves quantities such as mass and energy that are analytically conserved in the evolution equations. Whereas this does not guarantee that the initial data will be in perfect numerical hydrostatic balance, it is at least a physically motivated constraint and is sufficient for our simulations. The algorithm as described below is somewhat specific to our data set, but can be easily generalized to other cases of mapping 1D data to higher dimensions.

Our original 1D data is given as cells of known size (radius and mass) with known average values of intensive quantities (density, internal energy). Velocity is only given for the zone boundaries and hence total momentum conservation is somewhat arbitrary<sup>1</sup> as is kinetic energy conservation.

The first step is to construct a continuous function ( $C^0$ ) that conserves the physically conserved quantities. We found an ideal choice is to use “*volume coordinate*”,  $V$ , the volume enclosed by a given radius from the center of the star. We then use densities (mass density, energy density) such that the integral under the curve

$$X = \int_{V_1}^{V_2} \rho_X dV$$

is the total amount of the quantity  $X$  with space density  $\rho_X$  between the volume coordinates  $V_1$  and  $V_2$ .

Here we use a piecewise linear function that preserves the original monotonicity of the data (does not create artificial extrema) and is bounded by the original extrema of the data (Fig. 1). The scheme causes some “smearing” (smoothing) of the data which, however, is limited to less than one zone width of the original data. We use a two-step process: First we construct a linear interpolation across the interface between two zones which extends to the half-width of the smaller of the two zones. What is cut off from one zone by the interpolation function ( $a$  or  $b$ ) is added to the interpolation in the neighboring zone ( $a' = a$  and  $b' = b$ ). This would usually result

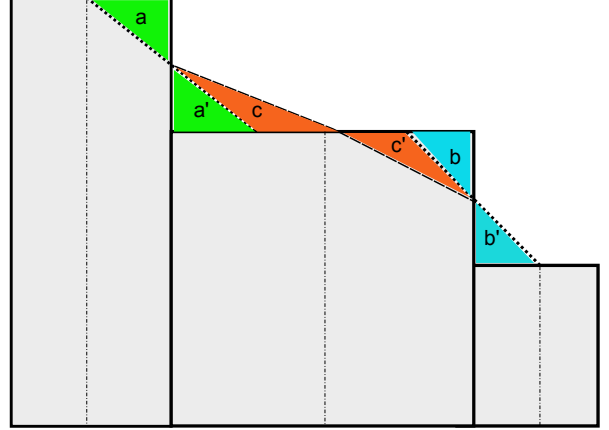


Figure 1: Schematic for constructing a conservative density profile.

in two “kinks” (change in slope) inside a zone (middle zone in Fig. 1) with a flat piece that usually is a poor approximation to the average gradient. We now correct the interpolation within each zone such that there is only one “kink” by finding a point in the flat piece such that we have equal area enclosed by the triangles on either side connecting from the new point to the value of the previous interpolation function at the boundaries of the zone ( $c = c'$ ).

In principle one could now integrate this function over the volume of the cell of the target grid to obtain the total amount of  $X$  within this cell. For the sake of generality of the interpolation function, we use an adaptive iterative sub-sampling method to obtain an acceptably converged integral: We evaluate the interpolation function at one point and multiply by the zone volume, then subdivide the zone and sample the function for each of these sub-volumes, multiply by the sub-volumes and sum up the result; this is repeated recursively for each sub-zone until the results from the last two levels agree to within the desired relative accuracy. Table 1 shows the results of mass deviation from original data after initial mapping by using linear interpolation and our algorithm; the number inside the () indicates the resolution of the mapped data. By comparing both results, our method has much higher accuracy and less depends on resolution and spacial dimensions.

This method is used to map internal energy density and the partial densities of the chemical species. The total density (or mass within the zone) is then obtained from the sum of partial densities (masses); pressure and temperature are obtained from the equation of state.

<sup>1</sup>since the initial model is spherically symmetric its total physical momentum is always zero for every radius bin

Table 1

Dimension	Linear Interpolation	Our Algorithm
1 D	2.91%(1024)	0.000254%(1024)
2 D	47.45%(1024 <sup>2</sup> )	0.005%(1024 <sup>2</sup> )
3 D	127.05%(128 <sup>3</sup> )	1.85%(2 <sup>3</sup> )

#### 4. Results and Discussion

We performed our simulations on Calhoun at the Minnesota Supercomputing Institute (MSI). The 2D run used about 4,000 CPUh; the 3D run used over 50,000 CPUh. The results presented here are from 2D runs of 150  $M_{\odot}$  PSN in cylindrical symmetry where we simulated only one hemisphere.

Figure 2a shows the specific nuclear energy generation rate in the inner  $2 \times 10^{10}$  cm domain at about 60 secs after maximum compression. The color coding shows the specific nuclear energy generation rate and in units of ergs/s/g on a logarithmic scale. We find Rayleigh-Taylor (RT) instabilities develop at the edge of the oxygen-burning shell. Later these instabilities will grow further and affect such properties as the observable light curve of the supernova. Figure 2b is a close-up of the RT instability.

#### 5. Conclusions

We have presented preliminary results from our first multidimensional numerical study of the evolution of pair instability supernova using the new Eulerian AMR radiation-hydrodynamics code CASTRO. We simulated the formation of Rayleigh-Taylor instabilities in the explosion of a 150  $M_{\odot}$  pair-instability supernova. We have introduced a new mapping method that can be used to define the initial conditions for multidimensional simulations from one-dimensional initial data in such a way that conservation of physical quantities, monotonicity, and continuity are guaranteed at any resolution.

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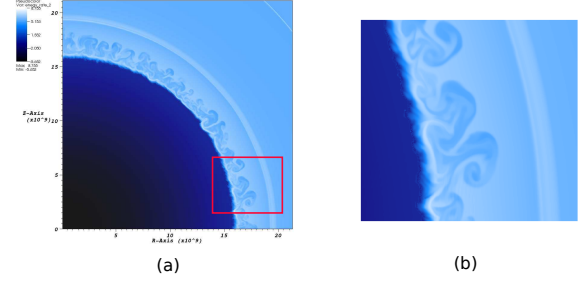


Figure 2: Energy generation rate

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